UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 5TH FEBRUARY 2009

Organised by the **United Kingdom Mathematics Trust**
and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

http://www.ukmt.org.uk

1. What is the value of $1 + 2^3 + 4 \times 5$?

A 27 B 29 C 55 D 65 E 155

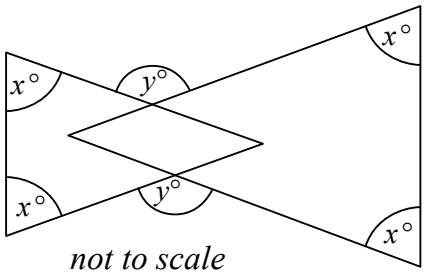
2. What is the sum of the first five non-prime positive integers?

A 15 B 18 C 27 D 28 E 39

3. Which of the following has the greatest value?

A 50% of 10 B 40% of 20 C 30 % of 30 D 20% of 40 E 10% of 50

4. The diagram shows two isosceles triangles, in which the four angles marked $x°$ are equal. The two angles marked $y°$ are also equal. Which of the following is always true?



not to scale

A $y = 2x$ B $y = x + 30$ C $y = x + 60$
D $y = x + 90$ E $y = 180 - x$

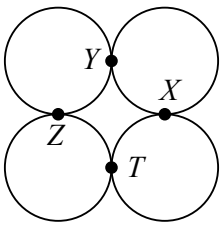
5. The square of a positive number is twice as big as the cube of that number. What is the number?

A 8 B 4 C 2 D $\frac{1}{2}$ E $\frac{1}{4}$

6. Which of the following is half way between $\frac{4}{5}$ and $-\frac{2}{3}$?

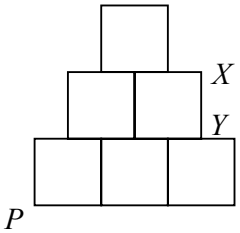
A $\frac{1}{15}$ B $\frac{7}{30}$ C $\frac{7}{15}$ D $\frac{17}{30}$ E $\frac{3}{4}$

7. Four touching circles all have radius 1 and their centres are at the corners of a square.
What is the radius of the circle through the points of contact X, Y, Z and T?



A $\frac{1}{2}$ B $\frac{1}{2}\sqrt{2}$ C 1 D $\sqrt{2}$ E 2

8. The diagram shows a figure made from six equal, touching squares arranged with a vertical line of symmetry. A straight line is drawn through the bottom corner P in such a way that the area of the figure is halved. Where will the cut cross the edge XY?



A at X B one quarter the way down XY
C half way down XY D three-quarters the way down XY
E at Y

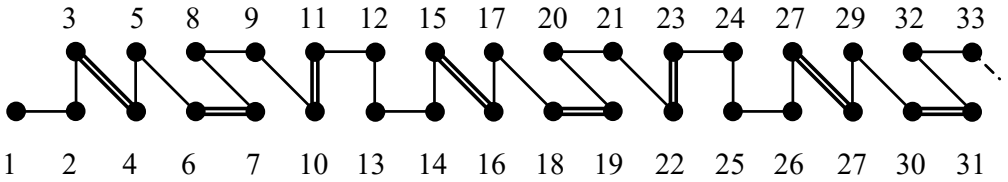
9. Joseph's flock has 55% more sheep than goats. What is the ratio of goats to sheep in the flock?

A 9:11 B 20:31 C 11:20 D 5:9 E 9:20

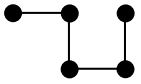
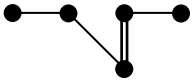
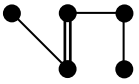
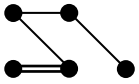
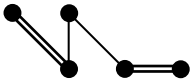
10. Fussy Fiona wants to buy a new house but she doesn't like house numbers that are divisible by 3 or by 5. If all the houses numbered between 100 and 150 inclusive are for sale, how many houses can she choose from?

A 24 B 25 C 26 D 27 E 28

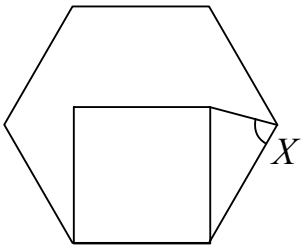
11. The diagram below shows a pattern which repeats every 12 dots.



Which of the following does the piece between 2007 and 2011 look like?

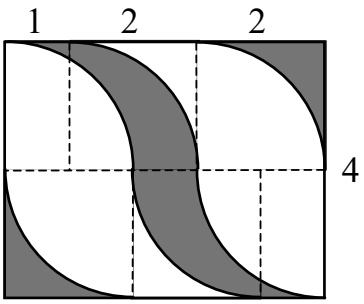
A  B  C  D  E 

12. The diagram shows a square inside a regular hexagon. What is the size of the marked angle at X?



A 45° B 50° C 60° D 75° E 80°

13. The diagram on the right shows a rectangle with sides of length 5 cm and 4 cm. All the arcs are quarter-circles of radius 2 cm. What is the total shaded area in cm^2?



A $12 - 2\pi$ B 8 C $8 + 2\pi$
D 10 E $20 - 4\pi$

14. Catherine's computer correctly calculates $\frac{66^{66}}{2}$. What is the units digit of its answer?

A 1 B 2 C 3 D 6 E 8

15. What is the value of $\frac{1}{x+2}$, given that $\frac{1}{x} = 3.5$?

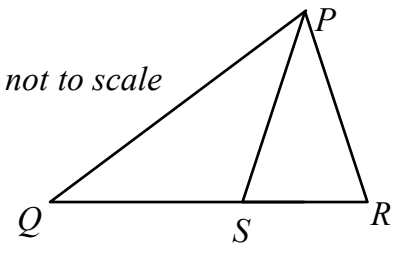
A $\frac{7}{9}$ B $\frac{7}{16}$ C $\frac{9}{7}$ D $\frac{7}{4}$ E $\frac{16}{7}$

16. How many different positive integers n are there for which n and $n^3 + 3$ are both prime numbers?

A 0 B 1 C 2 D 3 E infinitely many

17. PQR is a triangle and S is a point on QR.
$QP = QR = 9$ cm and $PR = PS = 6$ cm.
What is the length of SR?

A 1cm B 2cm C 3cm D 4cm E 5cm



18. If p, q are distinct primes less than 7, what is the largest possible value of the highest common factor of $2p^2q$ and $3pq^2$?

A 60 B 45 C 36 D 20 E 15

19. Driving to Birmingham airport, Mary cruised at 55 miles per hour for the first two hours and then flew along at 70 miles per hour for the remainder of the journey. Her average speed for the entire journey was 60 miles per hour. How long did Mary's journey to Birmingham Airport take?

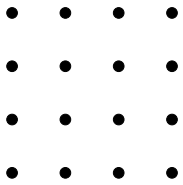
A 6 hours B $4\frac{1}{2}$ hours C 4 hours D $3\frac{1}{2}$ hours E 3 hours

20. A square, of side two units, is folded in half to form a triangle. A second fold is made, parallel to the first, so that the apex of this triangle folds onto a point on its base, thereby forming an isosceles trapezium. What is the perimeter of this trapezium?

A $4+\sqrt{2}$ B $4+2\sqrt{2}$ C $3+2\sqrt{2}$ D $2+3\sqrt{2}$ E 5

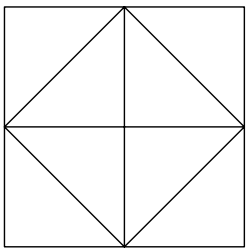
---

21. There are lots of ways of choosing three dots from this 4 by 4 array. How many triples of points are there where all three lie on a straight line (not necessarily equally spaced)?



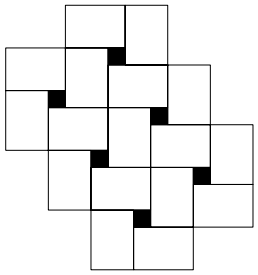
A 8 B 16 C 20 D 40 E 44

22. A square is divided into eight congruent triangles, as shown. Two of these triangles are selected at random and shaded black.
What is the probability that the resulting figure has at least one axis of symmetry?



A $\frac{1}{4}$ B $\frac{4}{7}$ C $\frac{1}{2}$ D $\frac{5}{7}$ E 1

23. The diagram shows part of a tiling pattern which is made from two types of individual tiles: 8 by 6 rectangular white tiles and square black tiles. If the pattern is extended to cover an infinite plane, what fraction is coloured black?



A $\frac{1}{12}$ B $\frac{1}{13}$ C $\frac{1}{25}$ D $\frac{1}{37}$ E $\frac{1}{40}$

24. What is the largest number of the following statements that can be true at the same time?

$0 < x^2 < 1$, $x^2 > 1$, $-1 < x < 0$, $0 < x < 1$, $0 < x - x^2 < 1$

A 1 B 2 C 3 D 4 E 5

25. One coin among N identical-looking coins is a fake and is slightly heavier than the others, which all have the same weight. To compare two groups of coins you are allowed to use a set of scales with two pans which balance exactly when the weight in each pan is the same. What is the largest value of N for which the fake coin can be identified using a maximum of two such comparisons?

A 4 B 6 C 7 D 8 E 9